Name: ________________________________

(Each problem is worth 20 points)

1. _________

2. _________

3. _________

4. _________

__________ (total score)
1. **Short answers.** For each question, give a short answer. No proof or explanation is required.

   a) If a square matrix is invertible, what can you say about its eigenvalues?

   b) If a square matrix is diagonalizable, what can you say about its eigenvectors?

   c) If a square matrix with real entries is symmetric, what can you say about its eigenvalues?

   d) If a square and symmetric matrix is indefinite (neither positive definite nor negative definite), what can you say about its eigenvalues?
2. **Transfer functions.** Consider a continuous-time system with the transfer function:

\[ Y(s) = \frac{1}{(s + 1)(s + 2)} U(s) \]

a) What is the impulse response of this system? Note: the Laplace transform of \( e^{-at} \) is \( \frac{1}{s+a} \).

b) Find a state-space realization for this system.

a) Compute the matrix exponential $e^{At}$, where $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

b) Compute the matrix exponential $e^{At}$, where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.
4. Controllability.

a) For which values of $\alpha$ is the following system controllable?

$$\dot{x}(t) = \begin{bmatrix} 1 & \alpha & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

b) We showed in class that if $(A, B)$ is not controllable and its controllability matrix has rank $q < n$, we can find state transformation matrix $T$ such that $(A, B) \rightarrow \left( \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ 0 & \hat{A}_{22} \end{bmatrix}, \begin{bmatrix} \hat{B}_1 \\ 0 \end{bmatrix} \right)$

where $\hat{A}_{11} \in \mathbb{R}^{q \times q}$. Prove that $(\hat{A}_{11}, \hat{B}_1)$ is controllable.