1. Introduction

- Logistics
- Examples
- About the course
- Detailed example with code
- First assignment

Laurent Lessard (www.laurentlessard.com)
Logistics

- Meeting times: Tue/Thu 1:00pm–2:15pm, Computer Science Building, Room 1221.

- Instructor: Laurent Lessard (laurent.lessard@wisc.edu)
  Office hours: Tue/Thu after class, location TBA

- Teaching Assistant: TBA

- Course website (notes, code samples, assignments): www.laurentlessard.com/teaching/524-intro-to-optimization/
The website will be updated frequently.
More logistics

• We will make extensive use of the online discussion forum at www.piazza.com. This is a great resource so take advantage of it! There will be homework hints and tips too.

• All assignments must be submitted **electronically**. We will use the website www.gradescope.com for all submissions and grading. I will sign you up and you should receive a confirmation email.
Prerequisites

- Exposure to a bit of linear algebra and calculus (Math 320, Math 340, or equivalent). We will review all relevant concepts as we need them.

- Comfortable with programming (CS 302 or equivalent). It doesn’t have to be Java, but if you’ve never written code in your life, this class will be very difficult for you.

- Very helpful (but not required): some experience with numerical computing (e.g. Matlab, Python, Julia, R)

For this course, we will exclusively use Julia. Julia is a new programming language for scientific computing similar to Matlab and Python. It’s open-source and super fast!
Coursework

- Homework (45%): Bi-weekly assignments, a mix of theory questions and practical problem-solving (with coding in Julia). Roughly 7 assignments total.

- Exam (25%): Thur Mar 16, 7:00pm–9:00pm, loc TBA. Mostly theory questions, covering the first 2/3 of the course (up until spring break).

- Final project (30%): You will work in groups of 3 or 4. More details to come...
There is no required textbook for the course. However, there are plenty of good references. Here are a few:


Ex. 1: Processor scheduling

- processor adjusts its speed $s_t$ over time
- jobs must be scheduled. Each job has: arrival time, deadline, total work required.
- energy consumed $\phi(s_t)$ depends on speed

Source: S. Boyd (Stanford)
Ex. 1: Processor scheduling

- uniform schedule: requires 204.3 energy.
- optimal schedule: requires 167.1 energy.
- similar to other types of scheduling problems! e.g. employees, power networks,...

Source: S. Boyd (Stanford)
Ex. 2: Fish and Wildlife Service

- Great lakes basin data visualization
- 250k interdependent barriers on the network of rivers
- Complex optimization for budget constraints, specific fish guilds, invasive species.

Source: M. Ferris (UW–Madison), Fishwerks.
**Ex. 3: Compressive sensing**

- $N$ outputs (observations) caused by $D$ inputs (features)
  - Disease prediction: $y$ are patients, $A$ are gene markers, or
  - Imaging: $y$ are image benchmarks, $A$ are MRI modalities.

System is under-determined!

Source: R. Baranyuk (Rice) et.al.
Ex. 3: Compressive sensing

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  - Disease prediction: $y$ are patients, $A$ are gene markers, or
  - Imaging: $y$ are image benchmarks, $A$ are MRI modalities.

System is over-determined! (Look for a sparse solution)

Source: R. Baranyuk (Rice) et.al.
Course objectives

1. Write down an algebraic formulation of an optimization problem that captures key elements of the problem. We call this an optimization model.

2. Perform standard mathematical manipulations and categorize the problem you’re dealing with.

3. Develop an intuition for different types of models, their solutions, and the implications and trade-offs involved.

4. Write code in Julia to solve these problems.

5. Learn to interpret solutions, validate, perform sensitivity analysis, explore trade-offs, etc.
Course topics (by model type)

1. Linear programs (LP)
2. Least squares and quadratic programs (QP)
3. Second-order cone programs (SOCP)
   (exam and spring break)
4. Integer programs (IP) and mixed-integer programs (MIP)
5. Nonlinear programs (NLP and MINLP)
   (final project due)
Categories of optimization models

We can categorize the model types from the previous slide:

- Linear vs. Nonlinear
- Convex vs. Nonconvex
- Discrete vs. Continuous
- Deterministic vs. Stochastic

These categorizations have a **significant** impact on the computational tractability of a problem instance.
Gateway to more advanced courses

- Linear programming (CS 525, 526)
- Convex analysis (CS 727)
- Stochastic/Dynamic programming (CS 719, 723)
- Integer Optimization (CS 728)
- Nonlinear optimization (CS 726, 730)

Selected applied topics:

- Machine learning (CS 760, 761, 762)
- Optimal control (ECE 719, 819, 821)
- Robot motion planning (ME 739, 780)
Top Brass example

Top Brass Trophy Company makes large championship trophies for youth athletic leagues. At the moment, they are planning production for fall sports: football and soccer. Each football trophy has a wood base, an engraved plaque, a large brass football on top, and returns $12 in profit. Soccer trophies are similar except that a brass soccer ball is on top, and the unit profit is only $9. Since the football has an asymmetric shape, its base requires 4 board feet of wood; the soccer base requires only 2 board feet. At the moment there are 1000 brass footballs in stock, 1500 soccer balls, 1750 plaques, and 4800 board feet of wood. What trophies should be produced from these supplies to maximize total profit assuming that all that are made can be sold?

football  soccer  both
## Top Brass data

### Recipe for building each trophy

<table>
<thead>
<tr>
<th></th>
<th>wood</th>
<th>plaques</th>
<th>footballs</th>
<th>soccer balls</th>
<th>profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>football</td>
<td>4 ft</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$12</td>
</tr>
<tr>
<td>soccer</td>
<td>2 ft</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$9</td>
</tr>
</tbody>
</table>

### Quantity of each ingredient in stock

<table>
<thead>
<tr>
<th></th>
<th>wood</th>
<th>plaques</th>
<th>footballs</th>
<th>soccer balls</th>
</tr>
</thead>
<tbody>
<tr>
<td>in stock</td>
<td>4800 ft</td>
<td>1750</td>
<td>1000</td>
<td>1500</td>
</tr>
</tbody>
</table>
Top Brass model components

1. Decision variables

- $f$: number of football trophies built
- $s$: number of soccer trophies built

2. Constraints

- $4f + 2s \leq 4800$ (wood budget)
- $f + s \leq 1750$ (plaque budget)
- $0 \leq f \leq 1000$ (football budget)
- $0 \leq s \leq 1500$ (soccer ball budget)

3. Cost function

- Maximize $12f + 9s$ (profit)
Top Brass model (optimization form)

maximize $f, s$

subject to: $4f + 2s \leq 4800$
$f + s \leq 1750$
$0 \leq f \leq 1000$
$0 \leq s \leq 1500$

- This is an instance of a linear program (LP), which is a type of optimization model.
- We have decision variables and parameters.
Top Brass model (generic)

maximize \( f, s \)

subject to:

\[
\begin{align*}
& a_{11} f + a_{12} s \leq b_1 \\
& a_{21} f + a_{22} s \leq b_2 \\
& \ell_1 \leq f \leq u_1 \\
& \ell_2 \leq s \leq u_2
\end{align*}
\]

• By changing the parameters, we create different instance.

• It’s good practice to separate parameters (data) from the algebraic structure (model).
Top Brass code (IJulia notebook)

Top Brass.ipynb

Note: we did not separate the data from the model in this example! Next class, we will see how we can improve the code.
Assignment #0 (not graded)

1. Get up and running with Julia + IJulia + JuMP.

2. Load the file Top Brass.ipynb in IJulia and confirm that you can reproduce the results shown in class.

3. Experiment! Try changing the parameters and seeing if the solution still makes sense.
Getting started with Julia

Two options:

1. Install Julia + IJulia + JuMP on your computer. Instructions [here](#). Might take a bit of tweaking to get things set up, but this is the most reliable way to run Julia.

2. Run Julia inside your browser at [https://www.juliabox.org/](https://www.juliabox.org/). Log in with any google account and create a new file using “Julia 0.5.0”. Once you’re in, install JuMP by typing in `Pkg.add("JuMP")` and pressing `SHIFT-ENTER`. This method is fastest, but probably less reliable in the long-run.
Julia tutorials


• Useful tutorial: https://learnxinyminutes.com/docs/julia/


• JuMP documentation: https://jump.readthedocs.org/en/latest/